An Analytical Solution for Transient Gas Flow in a Multi-Well System

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Soil vapor extraction (SVE) combined with air injection provides an efficient way for the cleanup of vadose zone contaminated by volatile organic chemicals (VOCs). A successful design of an SVE system, however, relies on a good knowledge of the induced gas flow field in the vadose zone. Analytical solutions are available to help understand the gas flow field at steady-state. However, most SVE systems must pass a transient period before reaching steady (or quasi-steady) state and the length of the period should be system-specific. This paper presents an analytical solution for transient gas flow in a vadose zone with extraction and injection wells. The transient solution approaches the steady-state solution as time increases. Calculations have shown that for a shallow well (screened in a depth of less than 10 m) in a vadose zone with an air permeability of 1 darcy (10^{-12} m^2) or larger, the system reaches steady-state in just several hours. Decreasing the air permeability or increasing the screen depth increases the time to reach steady-state. In practical applications the transient solution may be relatively insignificant in an SVE design. However, the solution can be important in site characterization through pneumatic tests. A procedure is provided for applying the dimensionless solution in estimating air permeability and air-filled porosity. An example is also given to use the transient solution for verifying numerical codes.

1. Introduction

In most cases, groundwater contamination is caused by spilling or leaking of volatile organic chemicals (VOCs) such as petroleum products and organic solvents. At these contaminated sites, a certain amount of VOC usually remains in the vadose zone and acts as a long-term source to groundwater contamination. It is impossible to clean a site without eliminating the contamination source in the vadose zone. In general, there are two ways to eliminate the contamination source in the vadose zone: soil excavation and in-situ remediation. For a shallow, accessible contamination source of relatively small extent, soil excavation is usually the choice because it is faster and cheaper. In other cases, we may need to leave the contaminated soil in its original place, and try the in-situ cleanup through biodegradation, chemical reaction, or soil vapor extraction and air injection. The extracted soil gas can carry VOC vapors away from the vadose zone, while the injected air can either help evaporate the VOC contaminants into the soil gas flow stream or deliver nutrients to the contamination source area for an enhanced biodegradation. To apply the methods of soil vapor extraction and air injection, designers need to locate the wells, determine the screen depths, and estimate the mass flow rates. All these decisions depend on prior knowledge on the induced gas flow field, which can only come from mathematical simulations.

Although sophisticated numerical codes are available for such kind of simulations, analytical solutions can easily provide a big picture of the induced gas flow under simplified conditions. In addition, analytical solutions can be used to verify numerical codes. Many analytical solutions for soil gas flow in vadose zone were developed in the past two decades. Some analytical solutions for simple radial flow can be borrowed from the corresponding solutions for groundwater. *McWhorter* [1990] presented an analytical solution for transient radial gas flow and applied it for estimating the gas permeability using pumping test data. *Shan* [1995] developed analytical solutions for transient, one-dimensional gas flow caused by barometric pumping, and applied these solutions to estimate the air permeability of the vadose zone using observed pressure variations at the land surface and at depths. *Shan et al.* [1999] also developed analytical solutions for transient, two-dimensional gas flow on a vertical vadose zone section, and presented methods for estimating the air permeability of a vertical leaky fault. For soil vapor extractions, useful analytical solutions are the ones for two-dimensional axisymmetrical flow in a vadose zone with the surface open to atmosphere. *Baehr and Hult* [1991] developed a solution for the case of a finite well radius and a thin soil layer at the surface. Their series solution is composed of terms containing Bessel functions. *Shan et al.* [1992] treated the extraction (or injection) well as a line sink (or source) and developed a series solution composed of terms of logarithm functions. Both solutions are for steady-state gas flow only. In addition to a solution for gas pressure, the later paper (*Shan et al.*, 1992) also provided the solution for the stream function. *Baehr and Joss* [1995] updated the solution of *Baehr and Hult* [1991] by improving the treatment on the upper boundary condition. There are many other analytical solutions for gas flow problems. Some examples are the ones for steady gas flow towards horizontal wells by *Falta* [1995], from air inlet wells by *Ross and Lu* [1994], and in a multi-well system by *Shan* [2006]. In a book-CD package, *Shan* [2004] selected 10 typical analytical solutions for one-, two- and three-dimensional gas flow in vadose zone, and programmed them in convenient Excel spreadsheet (Chapter 4, *Shan*, 2004). Illman and his coworkers used analytical solutions in the analyses of field data from pneumatic injection tests to estimate the air permeability and air-filled porosity (*Illman and Neuman*, 2000, and 2001, *Illman and Tartakovsky*, 2005a, b.) Here I want to add one more analytical solution to the literature: an analytical solution for transient gas flow in a multi-well system.

2. Theory

To simplify the problem, the vadose zone is homogeneous and has a uniform thickness, h [L]. Considering a vertical well screened at a depth interval of b [L] to a [L] ($0 < b < a < h$), the screen length, L is simply equal to $a - b$ (see Figure 1). For a given mass extraction (or injection) rate, m [M/T], the goals are to derive an analytical solution for transient gas flow induced by a single well, and to extend the solution to multi-well cases by means of the principle of superposition. Figure 1 shows the simplest multi-well system: the two-well case.

The following assumptions are made for further simplifying the problem: (a) the vadose zone is isotropic at least in the horizontal plane; (b) both temperature and atmospheric pressure remain constant in the process; (c) the extraction (or injection) rate is uniformly distributed over the screen interval; and (d) the soil water is immobile and there is no gas flow in the vadose zone initially. As in most cases air is the major component of soil gas, the properties of air are taken for those of soil gas. The coordinate is such chosen that the origin is at the land surface and the vertical (z) axis positive downwards (Figure 1).

For cases of a small pressure variation (30% or less), *Shan* and *Javandel* [1999] have shown that the pneumatic head is usually a better choice as the dependent variable. For this problem because the pressure change can be larger than 30%, the pressure (p [$M/L/T^2$]) squared is the appropriate choice for dependent variable. Following *Shan et al.* [1992] the governing equation is:

$$k_r \frac{\partial^2 p^2}{\partial x'^2} + k_r \frac{\partial^2 p^2}{\partial y'^2} + k_z \frac{\partial^2 p^2}{\partial z^2} = \frac{\varphi_a \mu}{P_a} \cdot \frac{\partial p^2}{\partial t} \tag{1}$$

where t [T] is time, (x', y', z) [L] the Cartesian coordinates, k_r [L^2] and k_z [L^2] the air permeability in the horizontal and vertical directions, respectively, φ_a the air-filled porosity, μ [M/L/T] the air viscosity, and P_a [M/L/T^2] the system mean pressure (for small pressure variation, the ambient pressure 1 atm = 101,325 Pa is usually taken as the mean pressure). In obtaining (1) the gravity effect on gas flow was neglected and the variable, p was replaced by a constant, P_a on the right-hand-side. In fact, such a simple anisotropic system can be converted to an isotropic system by introducing the following transform:

$$x = \sqrt{\frac{k_z}{k_r}} x'; \qquad y = \sqrt{\frac{k_z}{k_r}} y' \tag{2}$$

The governing equation (1) after the transform is simplified into:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{\alpha} \cdot \frac{\partial u}{\partial t} \tag{3}$$

The new variable u and parameter α (called *pneumatic diffusivity*) are defined by:

$$u = p^2 - P_a^2 \tag{4a}$$

$$\alpha = \frac{P_a k_z}{\varphi_a \mu} \tag{4b}$$

By the above assumptions the initial condition is:

$$u(x, y, z, 0) = 0 \tag{5}$$

The boundary conditions are:

$$u(x, y, 0, t) = 0 \tag{6a}$$

$$\left(\frac{\partial u}{\partial z} \right)_{z=h} = 0 \tag{6b}$$

Here a Cartesian coordinate is chosen for obtaining a solution that is applicable to multi-well cases. For the same purpose the well is set at an arbitrary location, (x_w, y_w). By taking an infinitely small increment dz_w at a point z_w in the well-screen interval ($b < z_w < a$), point z_w can be treated as a point source (or sink) with a strength of mdz_w/L. Following that of Carslaw and Jaeger (1959, page 261), the solution of (3) for the point source (or sink) with an initial condition represented by (5) in an infinite medium is:

$$du = c \cdot \frac{erfc[\sqrt{r^2 + (z - z_w)^2} / \sqrt{4\alpha t}\,]}{\sqrt{r^2 + (z - z_w)^2}} dz_w \tag{7}$$

Here $erfc(x)$ is the complementary error function; the radial distance, r and the coefficient, c are defined by:

$$r = \sqrt{(x - x_w)^2 + (y - y_w)^2} \tag{8a}$$

$$c = \frac{P_a RTm}{2\pi\alpha\varphi_a L} = \frac{\mu RTm}{2\pi k_z L} \tag{8b}$$

In (8b), R [$L^2/T^2/K$] is the gas constant for air (287 $m^2/s^2/K$), and T [K] the temperature. However, the vadose zone is not an infinite medium. Instead, it is bounded above by a constant pressure boundary, and below by a no flow boundary (the groundwater table). The solution that satisfies boundary conditions (6a) and (6b) can be obtained by using the method of images (Shan, et al., 1992):

$$du = cdz_w \sum_{n=0}^{\infty} \left[\frac{erfc[\sqrt{r^2 + (z - z_n^{+})^2} / \sqrt{4\alpha t}\,]}{\sqrt{r^2 + (z - z_n^{+})^2}} - \frac{erfc[\sqrt{r^2 + (z - z_n^{-})^2} / \sqrt{4\alpha t}\,]}{\sqrt{r^2 + (z - z_n^{-})^2}} \right] \tag{9}$$

where the locations of the images are calculated by (Shan, et al., 1992):

$$z_n^{+} = (-1)^n z_w \pm 2nh \qquad (n = 0, 1, 2, \cdots) \tag{10a}$$

$$z_n^{-} = (-1)^{n+1} z_w \pm 2nh \qquad (n = 0, 1, 2, \cdots) \tag{10b}$$

Equation (9) is the solution for a continuous point source (or sink). The solution for a continuous line source (or sink) is simply the sum of (9) for all point sources (or sinks) as z_w varies from b to a, i.e.

$$u = c\sum_{n=0}^{\infty}\int_{b}^{a} dz_w \left[\frac{erfc[\sqrt{r^2+(z-z_n{}^{+})^2}/\sqrt{4\alpha t}]}{\sqrt{r^2+(z-z_n{}^{+})^2}} - \frac{erfc[\sqrt{r^2+(z-z_n{}^{-})^2}/\sqrt{4\alpha t}]}{\sqrt{r^2+(z-z_n{}^{-})^2}} \right] \tag{11}$$

Because (11) is the solution for a well at an arbitrary location, it can be extended to the solution for a multi-well system using the principle of superposition:

$$u = \sum_{i=1}^{N} u_i \tag{12}$$

Here N is the total number of wells, and u_i the change of pressure squared due to the activity at the ith well, which can be calculated using (11). In each calculation for u_i, one should use the corresponding well parameters such as m_i, L_i, a_i, b_i, x_{wi}, and y_{wi}. The mass rate, m_i is positive for injection and negative for extraction.

The solutions in (11) and (12) both have the dimension of pressure-squared, which may be useful in predicting pressure variations. In site characterization where air permeability and air-filled porosity are unknown, a dimensionless solution is much more useful. To obtain a dimensionless solution, I take the thickness of vadose zone (h) as the characteristic length and introduce the following dimensionless variables:

$$u_D = u/c; \qquad t_D = \alpha t/h^2; \tag{13a}$$

$$l_D = l/h \qquad (l = a, b, r, z, z_w, z_n^{+}, z_n^{-}). \tag{13b}$$

The applications of (13a) and (13b) to (11) lead to the following dimensionless solution:

$$u_D = \sum_{n=0}^{\infty}\int_{b_D}^{a_D} dz_{wD} \left[\frac{erfc[\sqrt{r_D^2+(z_D-z_{nD}{}^{+})^2}/(2\sqrt{t_D})]}{\sqrt{r_D^2+(z_D-z_{nD}{}^{+})^2}} - \frac{erfc[\sqrt{r_D^2+(z_D-z_{nD}{}^{-})^2}/(2\sqrt{t_D})]}{\sqrt{r_D^2+(z_D-z_{nD}{}^{-})^2}} \right]$$

(14)

3. Results

For convenience, the parameters listed in Table 1 of *Shan et al.* [1992] were used for example calculations For simplification, the vadose zone was assumed isotropic such that $k_r = k_z = k$. Table 1 shows all default parameters used in the following calculations, for both single-well and multi-well systems. The only exception is that for the multi-well system, a different mass rate will be applied at the injection well. Although (11) and (14) are both series solutions containing infinite number of terms, they converge very fast. Calculations have shown that the first 20 terms usually gives sufficiently accurate results. Two verifications are conducted as follows before the demonstration of potential applications.

3.1. Verifications

The first verification is a comparison of the transient solution for a single well with the corresponding steady-state analytical solution (*Shan et al.*, 1992). The verification can be done in two ways: by inspecting solution (11), or by comparing the numerical results.

As time t tends to infinity, the two numerators in (11) both approach the limit of unity because $erfc(0) = 1$, which reduces (11) into the steady-state solution by *Shan et al.*, (1992). A comparison of the transient solutions with the steady-state solution is shown in Figure 2, which was calculated using a = 7 m, b = 3m, h = 10 m, $k = 10^{-12}$ m^2, and m = -0.025 kg/s. Two transient pressure profiles were calculated at the depth of 5 m and at two different times: 0.1 and 10 hours. These profiles were compared with the one calculated by the steady-state solution (*Shan et al.*, 1992). Figure 2 shows that the early time (t = 0.1 hours) pressure profile (the dashed line) is quite different from the steady-state one (the solid line), but the late time (t = 10 hours) profile (the solid dots) is almost the same as the steady-state solution (the solid line).

The second verification is applying the analytical solution to verify a well-developed numerical code. I choose TOUGH2 (*Pruess*, *et al.*, 1999), a numerical code in the public domain for two main reasons: a) it has been verified against many analytical solutions in its decades-history of development, and b) one of its modules, EOS3 can easily perform the task. Although the module EOS3 was originally designed to simulate two-phase (water and gas) flow problem, one can always turn off one phase (the water phase in this study) by specifying an initial phase saturation that is much smaller than its residual saturation (i.e., to make the soil water immobile).

The case of a shallow well (a = 7 m & b = 3m) in a very permeable vadose zone (h = 10 m and $k = 10^{-11}$ m^2) was taken for simulation. The 10 m vadose zone was divided into 51 rows. The top and bottom rows were both 0.1 m high, and the rest 49 rows 0.2 m high. The grid size in the radial direction was varied as follows: the first column (on a vertical section) had a radius of 0.1 m, the second to 99th column a uniform grid size (Δr) of 0.2 m. The Δr was 1 m for Column # 100, and 5 m for Columns # 101 to 110. The radius of the model domain is 70.7 m. The top of the model domain (the land surface) maintained the ambient pressure, and the bottom and the perimeter of the model domain were no-flow boundaries. Although the boundary condition at the perimeter did not match that of the analytical solution, the error caused to the results should be minimum because the model boundary (at 70.7 m) was far from the points of calculation, and the simulation time was very small (1 hour). This was confirmed by the very small pressure change at the elements close to the boundary at the end of simulation.

Figure 3 shows the comparison of the analytical solution with the TOUGH2 solution at the depth of 5 m and two different radial distances: 5 m and 10 m. The two solid lines represent the analytical solutions, and the solid dots represent the TOUGH2 solutions. As the analytical solution neglected the gravity effect, for comparison, the gravity effect was purposely turned off in a TOUGH2 simulation. The results are shown in Figure 3 as circles. As shown in Figure 3, the analytical and numerical solutions do not match very well. Two possible causes for the differences are: small boundary or mesh effect in the numerical modeling, and the approximations in deriving the analytical solution. Despite of the possible small errors, the analytical solution provides a convenient and sufficiently accurate tool for field studies.

3.2. Single-Well System

The single well solutions have potential applications in two ways: a direct application for determining system parameters (e.g., the extraction rate and well screen depths) using (11), and an inverse application for estimating soil properties using (14). For a single well case, the z-axis is set at the well such that $x_w = y_w = 0$, and that the gas flow is symmetrical with respect to the z-axis. In such a simple coordinate, r is the radial distance from the calculation point to the extraction well.

The results of two examples are shown in Figures 4, and 5, respectively. In both examples, the air permeability was varied from 10^{-12} to 10^{-11} m^2 and the pressure variations were calculated at two radial distances: 5 and 10 m. In Figure 4, the thickness of the vadose zone is 10 m, the well is screened in the depth interval of 3 to 7 m, and the pressure variations are calculated at the depth of 5 m. The case is called “the shallow-well case”. In Figure 5, the thickness of the vadose zone is 30 m, the well is screened in the depth interval of 23 to 27 m, and the pressure variations are calculated at the depth of 25 m. The case is called "the deep-well case". A comparison of the pressure variation curves in Figures 4 and 5 indicates that the air permeability has a dominant impact on the magnitude of pressure variation and the time to reach steady. Where air permeability is sufficiently large (10^{-11} m^2 or 10 darcy), the pressure variations in both cases are small (0.025 atm or less), and the pressure reaches steady-state in less than one hour. As the air permeability decreases to 10^{-12} m^2 (or 1 darcy), the effect of the length of air-flow path becomes significant. As a result, the maximum pressure variation is larger than 0.15 atm, and the time to reach steady-state is larger than three hours (Figure 4). Figure 5 shows that both the pressure drop and the time to reach steady-state in the deep-well case are much larger than those of the shallow-well case (Figure 4). Obviously, it is important to accurately estimate the air permeability of the vadose zone.

The dimensionless solution (14) can be applied to estimate the air permeability and the air-filled porosity by means of curve-fitting against field test data. I recommend the following procedures for such an application.

Step 1 Use (13b) to calculate the dimensionless depth-interval b_D and a_D of the extraction well, and the dimensionless coordinates (r_D, z_D) of the observation point.

Step 2 Use (14) to calculate the corresponding type curve such as the one in Figure 6 ($a_D = 0.7$, $b_D = 0.3$, and $r_D = z_D = 0.5$). Plot the curve in a log-log coordinate.

Step 3 Plot the observation data, u vs. t in a log-log coordinate.

Step 4 Fit the observation data (e.g., the circles in Figure 6) with the type curve (e.g., the solid line in Figure 6) and choose a point (such as P in Figure 6).

Step 5 Read the coordinates of the point (such as P in Figure 6) in the two coordinate systems.

Step 6 Use the following formulae to calculate the air permeability and the air-filled porosity.

$$k_z = \frac{\mu R T m u_D}{2\pi L u} \tag{15a}$$

$$\varphi_a = \frac{P_a k_z t}{\mu h^2 t_D} \tag{15b}$$

The formulae were simply derived from (4b), (8b) and (13a).

Here the subscript, z for the air permeability was retained for an anisotropic system, where a perfect curve-fitting cannot be reached on Step 4. In that case, one can vary r_D and recalculate the type curve to achieve a best-fit. The ratio of the best-fit r_D and the original r_D (r/h) is the square-root of the permeability ratio given in (2).

In the above procedure, one should ignore the sign for mass rate (m) and treat u as a positive value. In other words, when calculating u using (4a), one should always take its absolute value. In the example (Figure 6), $t_D = 0.1$ and $u_D = 0.2$; $t = 600$ s and $u = 1.14 \times 10^9$ Pa^2 (from the observation-data coordinate not shown in Figure 6;) $h = 10$ m, $L = 4$ m, $m = 0.1$ kg/s, $T = 10$ °C = 283 K, and $\mu = 1.76 \times 10^{-5}$ kg/m/s. Substituting all the parameters and $R = 287$ m^2/s^2/K into (15a) and (15b), we estimated: $k_z \approx 10^{-12}$ m^2, and $\varphi_a \approx 0.345$. Here the best-fit was achieved at the original r_D, which means that the vadose zone is isotropic.

3.3. Multi-Well System

A hypothetical two-well system in a 30 m thick, less permeable vadose zone ($k = 10^{-12}$ m^2) was chosen for demonstration. An extraction well was screened in the depth interval of 3 to 7 m, and an injection well in the depth interval of 23 to 27 m. The extraction rate was 0.1 kg/s, and the injection rate 0.05 kg/s. The distance between the two wells was 20 m. The y-coordinate was set to pass through the center of the two wells (see Figure 7). In such a coordinate system, the locations of the wells and their screens were given in Table 3. Other default parameters used in the two-well calculations were given in Table 1. The pressure variations were calculated at four points: A (0, 0, 5), B (0, 0, 15), C (0, 0, 25), and D (10, 0, 25). The locations for points A, B, and C were shown in Figure 7. Point D was actually 10 m away from Point C and the y-z plane. As shown in Figure 8, the pressures at points A and B decreased at the beginning and gradually increased later. However, the pressures at points C and D never decreased but started increase rapidly after a short while. Points A and B were closer to the extraction well and thus only got affected by the injection well at later time. Except for point A that was very close to the extraction well, all three points eventually reached certain steady-state pressures that were higher than the ambient pressure (Figure 8).

4. Conclusions

A well-designed soil vapor extraction system can efficiently cleanup the VOC contamination in the vadose zone, reducing cost and saving time. Such a design relies heavily on mathematical simulations by either numerical codes or analytical solutions. Previously available analytical solutions for soil gas flow in the vadose zone were usually derived based on the assumption of a steady-state. An analytical solution for transient gas flow in a vadose zone is developed to validate the assumption under different field conditions. For cases of a shallow well in a very permeable vadose zone, the time to reach steady-state can be several hours only. Increasing the well depth and decreasing the air permeability can both significantly increase the time to reach steady-state. Example calculations have shown that transient gas flow may be important at sites where the vadose zone is less permeable and the well screen is relatively deep. A step-by step procedure is provided for the application of a dimensionless single-well solution to estimate the air permeability and air-filled porosity. The multi-well solution should be useful in the performance study on a system containing extraction and injection wells. Like all other analytical solutions, the solution is useful for verifying numerical codes. It should be noted that: a) the approximation of variable, p by the ambient pressure, P_a in the process of linearizing the governing equation tends to slightly overestimate the pressure under extraction conditions; and b) the neglect of gravity tends to slightly underestimate pressure under extraction conditions. The impacts of these assumptions to calculated pressures under injection condition should be the opposite. The errors caused by the two assumptions can cancel out each other at some locations, and in some mixed extraction-injection systems.

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Tables

Table 1. Default parameters for example calculations.

Mass rate (m)	– 0.1 kg/s
Air-filled porosity (n)	0.4
Ambient pressure (P_a)	1 atm =101,325 Pa
Temperature (T)	10^oC
Air viscosity (μ)	1.76×10^{-5} kg/m/s

Table 2. Varying parameters for sensitivity studies using a single well.

Vadose zone thickness (h)	10 m (shallow)	30 m (deep)
Depth to screen bottom (a)	7 m (shallow)	27 m (deep)
Depth to screen top (b)	3 m (shallow)	23 m (deep)
Air permeability (k)	10^{-11} m^2 (very permeable)	10^{-12} m^2 (less permeable)

Table 3. Well parameters for two-well studies.

Parameters	Well 1	Well 2
Well x-coordinate (x_w)	0	0
Well y-coordinate (y_w)	-10 m	10 m
Depth to screen top (b)	3 m	23 m
Depth to screen bottom (a)	7 m	27 m
Mass rate (m)	-0.1 kg/s	0.05 kg/s